Electromagnetic Fields / Fundamentals (ELE242)(CCE302)

Chapter (02)
Review on Vectors & Coordinates

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Outline

1-Scalar and Vector Quantities

2- Dot and Cross Products

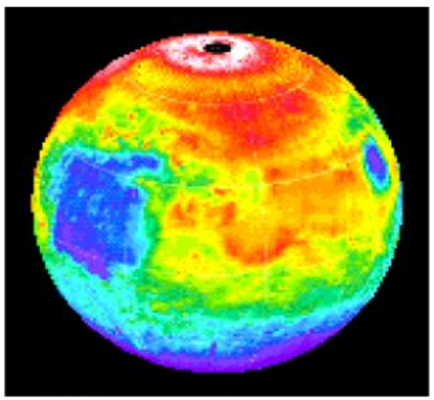
3- Coordinate Systems

- Cartesian Coordinates
- Cylindrical Coordinates
- Spherical Coordinates

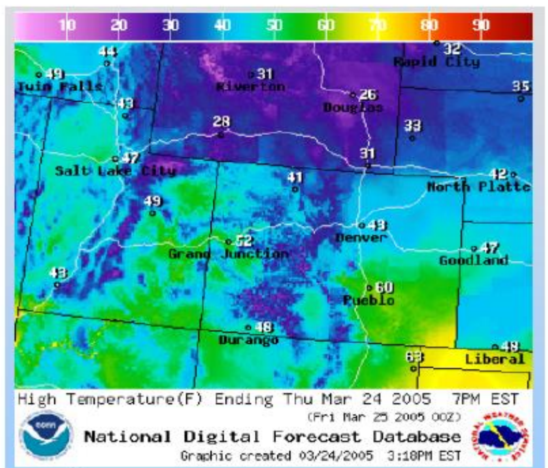
Scalars and Vectors

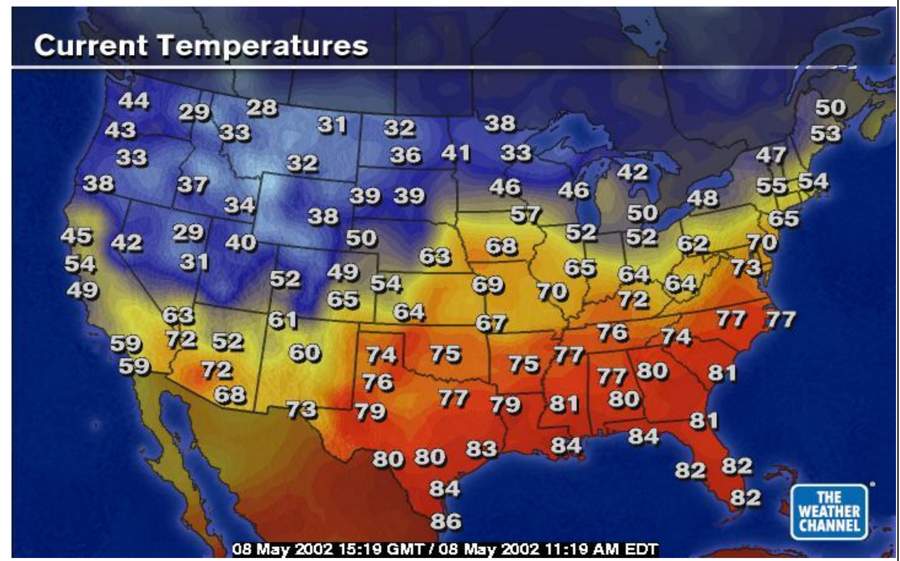
- Scalar refers to a quantity whose value may be represented by a single (positive or negative) real number.
- Some examples include distance, temperature, mass, density, pressure, volume, and time.

Temperature: Every location has associated value (number with units)



Nighttime temperature map for Mars





Scalars and Vectors

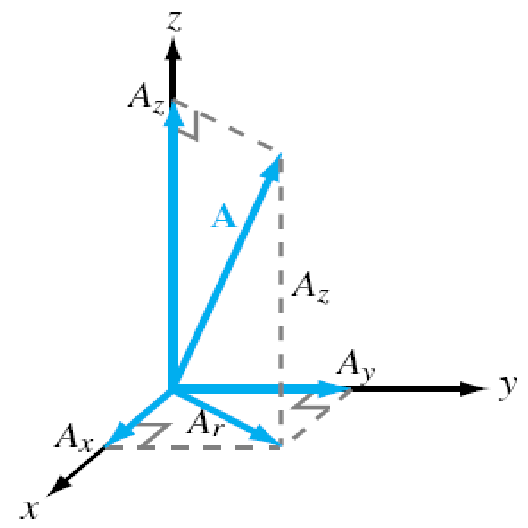
- A vector quantity has both a magnitude and a direction in space. We especially concerned with two- and three-dimensional spaces only.

- Displacement, velocity, acceleration, and force are examples of vectors.

- Scalar notation: *A* or A (*italic* or plain)
- Vector notation: **A** or $\vec{A}$ (bold or plain with arrow)

Vector in Cartesian Coordinates

A vector $\vec{A}$ in Cartesian Coordinates may be represented by three component vectors, which are A_x, A_y and A_z.



$$\vec{A} = \left(A_x, A_y, A_z\right)$$

OR

$$A = \hat{x}A_x + \hat{y}A_y + \hat{z}A_z$$

The magnitude of A is written as |A|:

$$|A| = \sqrt{A_x{}^2 + A_y{}^2 + A_z{}^2}$$

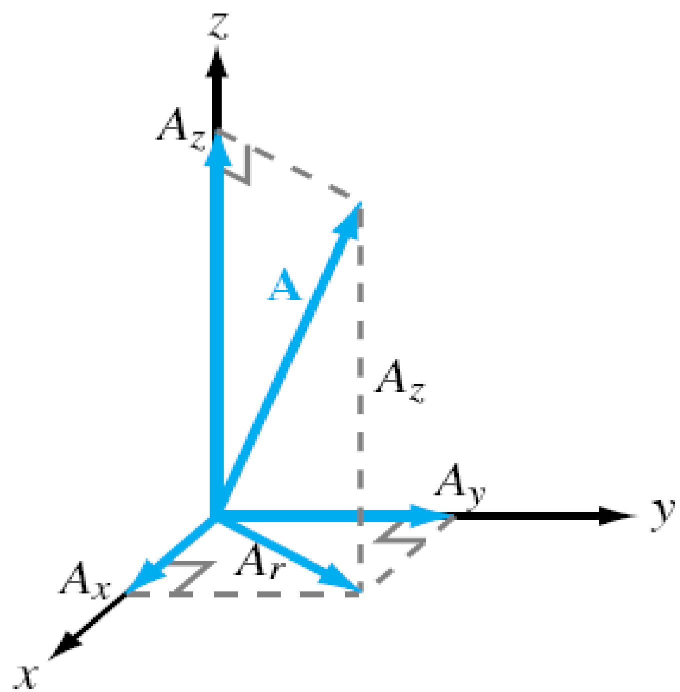
Unit Vectors

- A unit vector along vector **A** is;
  - A vector with magnitude = 1 (unity)
  - Directed along the coordinate axes in the direction of increasing coordinate values

Unit Vectors

Unit vector in the direction of vector **A** is

$$a_A = \frac{\vec{A}}{\sqrt{A_x^{\ 2} + A_y^{\ 2} + A_z^{\ 2}}} = \frac{\vec{A}}{|A|}$$



Example 1: Unit Vector

Specify the unit vector extending from the origin towards the point

$$G(2,-2,-1)$$

- Construct the vector extending from origin to point G

$$\mathbf{G} = 2\hat{x} - 2\hat{y} - \hat{z}$$

- Find the magnitude of $\vec{\mathbf{G}}$

$$|\mathbf{G}| = \sqrt{(2)^2 + (-2)^2 + (-1)^2} = 3$$

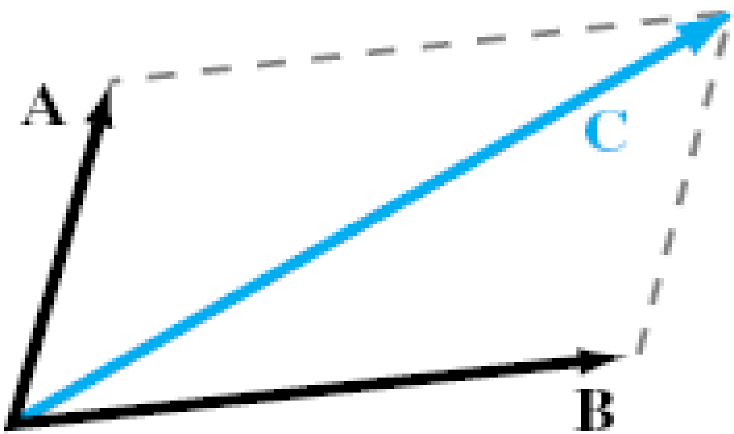
$$\mathbf{a}_G = \frac{\vec{\mathbf{G}}}{|\mathbf{G}|} = \frac{2}{3}\hat{x} - \frac{2}{3}\hat{y} - \frac{1}{3}\hat{z}$$

$$= 0.667\hat{x} - 0.667\hat{y} - 0.333\hat{z}$$

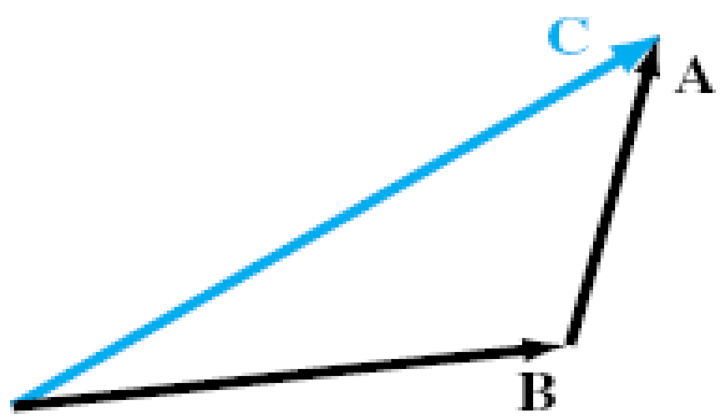
Vector Algebra

- For addition and subtraction of **A** and **B**,

$$\mathrm{C} = \mathrm{A} + \mathrm{B} = \hat{x}\left(A_x + B_x\right) + \hat{y}\left(A_y + B_y\right) + \hat{z}\left(A_z + B_z\right)$$
$$\mathrm{D} = \mathrm{A} - \mathrm{B} = \hat{x}\left(A_x - B_x\right) + \hat{y}\left(A_y - B_y\right) + \hat{z}\left(A_z - B_z\right)$$



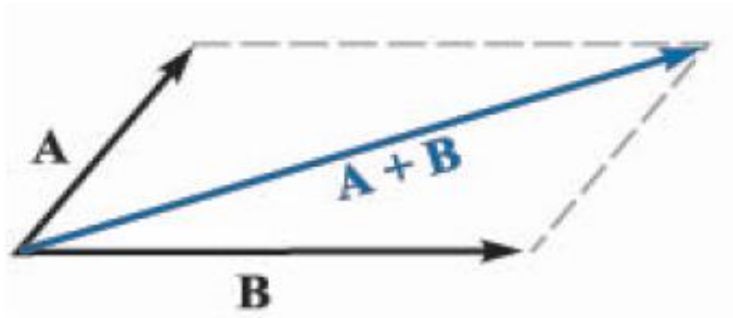
(a) Parallelogram rule

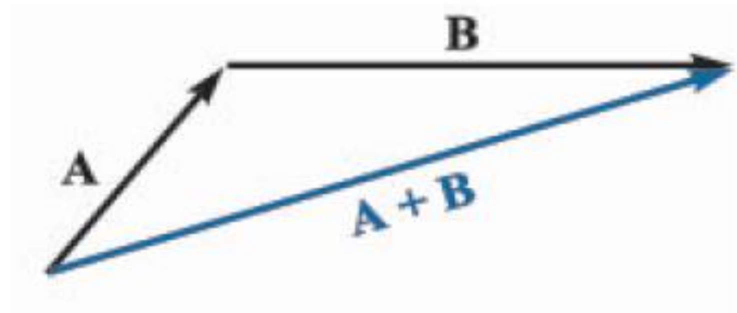


(b) Head-to-tail rule

Vector Algebra

Two vectors may be added graphically either by:





1- Drawing both vectors from a common origin and completing the parallelogram

2- By beginning the second vector from the head of the first and completing the triangle; either method is easily extended to three or more vectors

Vector Algebra

Commutative Law $$\mathbf{A}+\mathbf{B}=\mathbf{B}+\mathbf{A}$$

Associative Law $$\mathbf{A}+(\mathbf{B}+\mathbf{C})=(\mathbf{A}+\mathbf{B})+\mathbf{C}$$

$$\mathbf{A}-\mathbf{B}=\mathbf{A}+(-\mathbf{B})$$

$$\frac{\mathbf{A}}{n}=\frac{1}{n}\mathbf{A}$$

$$\mathbf{A}-\mathbf{B}=0\rightarrow\mathbf{A}=\mathbf{B}$$

Example 2: Vector Algebra

If $\vec{A} = 10\hat{x} - 4\hat{y} + 6\hat{z}$

$\vec{B} = 2\hat{x} + \hat{y}$

Find:

(a) The component of $\vec{\mathbf{A}}$ along $\hat{y}$

(b) The magnitude of $3\vec{\mathbf{A}} - \vec{\mathbf{B}}$

(c) A unit vector $\vec{\mathbf{C}}$ along $\vec{\mathbf{A}} + 2\vec{\mathbf{B}}$

Solution to Example 2

$$\vec{A} = 10\hat{x} - 4\hat{y} + 6\hat{z} \qquad \vec{B} = 2\hat{x} + \hat{y}$$

(a) The component of $\vec{\mathbf{A}}$ along $\hat{y}$ is

$$A_y = -4$$

(b) $$\left|3\vec{A} - \vec{B}\right| = ?$$

$$\begin{aligned} 3\vec{A} - \vec{B} &= 3(10,-4,6) - (2,1,0) \\ &= (30,-12,18) - (2,1,0) \\ &= (28,-13,18) \\ &= 28\hat{x} - 13\hat{y} + 18\hat{z} \end{aligned}$$

Solution to Example 2

Hence, the magnitude of $3\vec{\mathbf{A}} - \vec{\mathbf{B}}$ is:

$$\left|3\mathbf{A} - \mathbf{B}\right| = \sqrt{(28)^2 + (-13)^2 + (18)^2} = 35.74$$

(c) A unit vector $\vec{\mathbf{C}}$ along $\vec{\mathbf{A}} + 2\vec{\mathbf{B}}$

Let

$$\begin{aligned} \vec{\mathrm{C}} &= \vec{\mathrm{A}} + 2\vec{\mathrm{B}} \\ &= (10,-4,6) + (4,2,0) \\ &= (14,-2,6) \\ &= 14\hat{\mathrm{x}} - 2\hat{\mathrm{y}} + 6\hat{\mathrm{z}} \end{aligned}$$

Solution to Example 2

So, the unit vector along $\vec{\mathbf{C}}$ is:

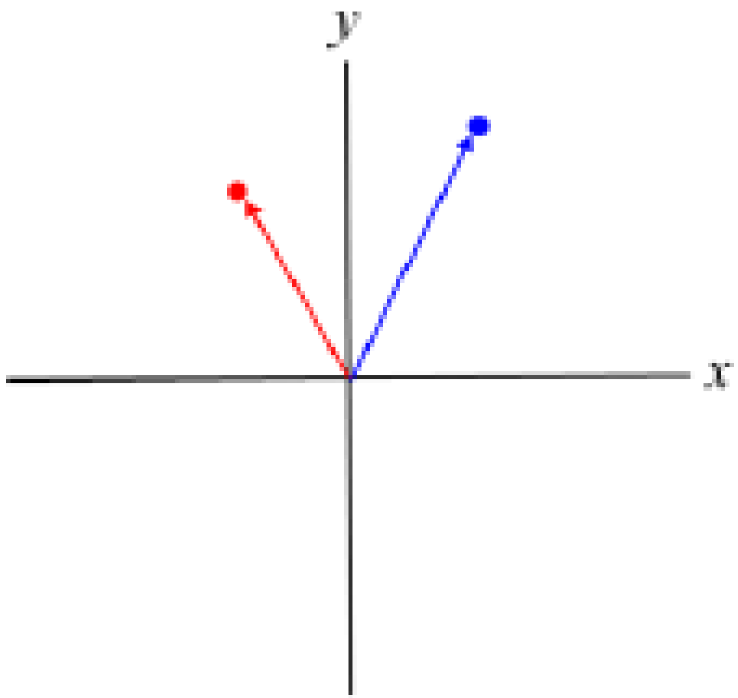
$$\hat{\mathbf{c}} = \frac{\vec{\mathbf{C}}}{\left|\mathbf{C}\right|} = \frac{(14,-2,6)}{\sqrt{(14)^2 + (-2)^2 + (6)^2}}$$

$$= \frac{14}{15.36}\hat{\mathrm{x}} - \frac{2}{15.36}\hat{\mathrm{y}} + \frac{6}{15.36}\hat{\mathrm{z}}$$

$$= 0.911\hat{\mathrm{x}} - 0.130\hat{\mathrm{y}} + 0.391\hat{\mathrm{z}}$$

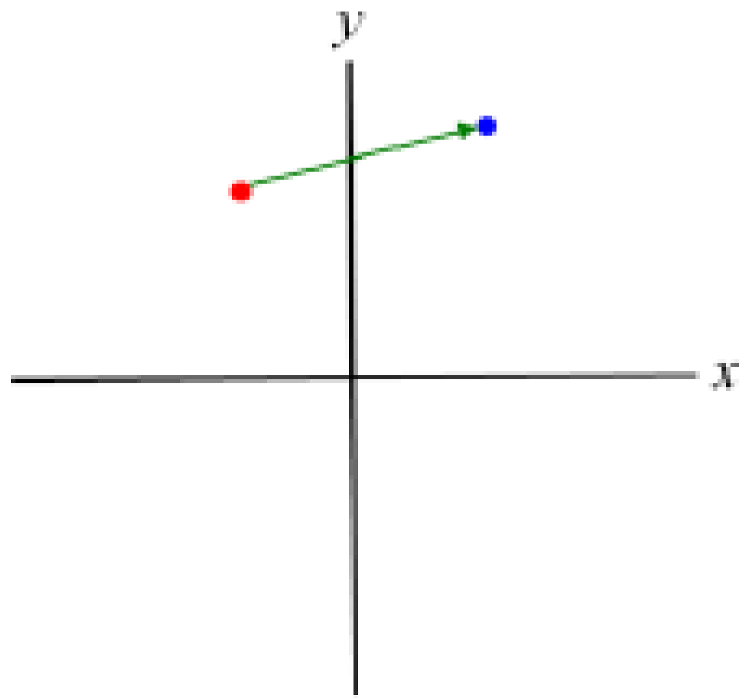
Position and Distance Vectors

Position



Position relative to the origin.

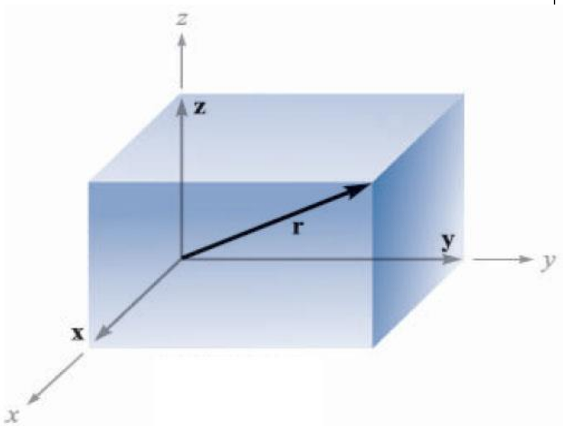
Distance



Vectors can indicate distance, but the origin is not given.

Position and Distance Vectors

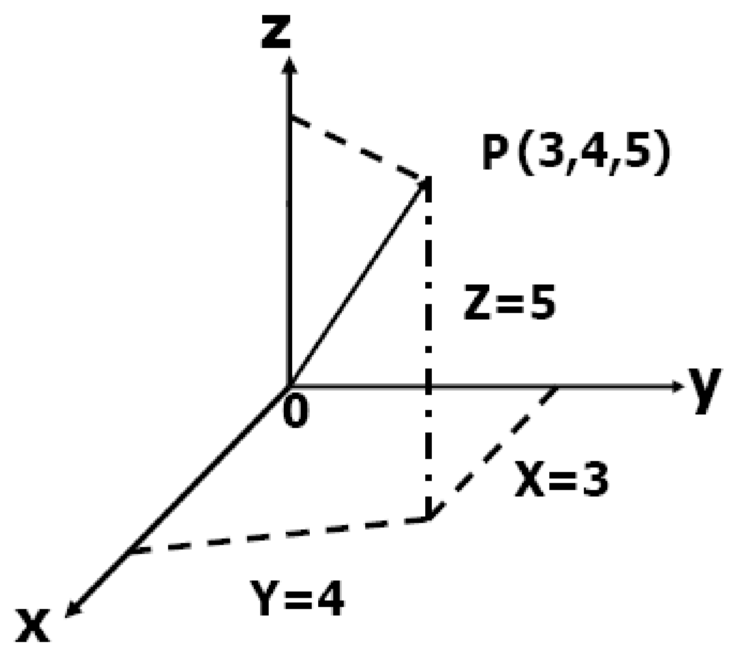
- A point P in Cartesian coordinate maybe represented as $\mathbf{P}(x, y, z)$



- The position vector $\vec{\mathbf{r}}_P$ of point P is the vector to point P

$$\vec{\mathbf{r}}_P = OP = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z$$

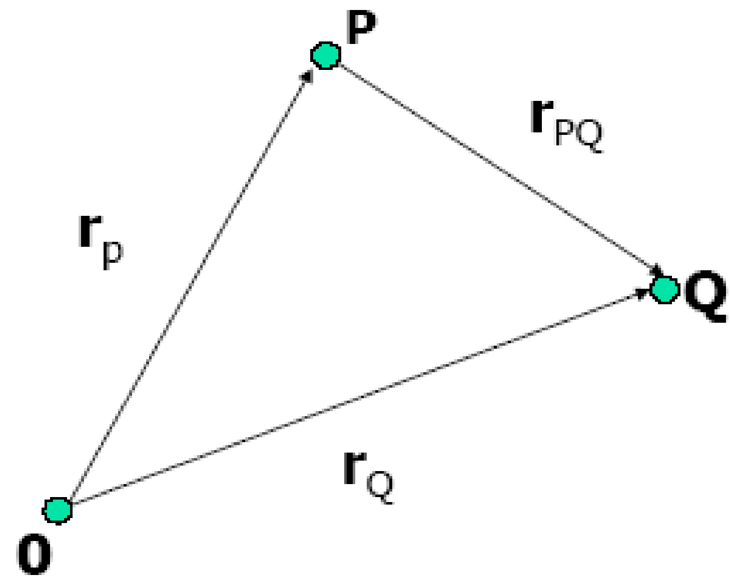
$$\vec{\mathbf{r}}_P = 3\mathbf{a}_x + 4\mathbf{a}_y + 5\mathbf{a}_z$$



Position and Distance Vectors

If we have two position vectors, $\vec{\mathbf{r}}_Q$ and $\vec{r}_P$, the third vector or *distance vector* can be defined as:-

$$\vec{\mathbf{r}}_{PQ} = \vec{\mathbf{r}}_Q - \vec{\mathbf{r}}_P$$



Example 3: Position Vectors

Point P and Q are located at $(0,2,4)$ and $(-3,1,5)$. Calculate:

(a) The position vector P

(b) The distance vector from P to Q

(c) The distance between P and Q

Solution to Example 3

(a) The position vector of point P $\quad (0,2,4)$

$$\vec{\mathbf{r}}_P = 0\mathbf{a}_x + 2\mathbf{a}_y + 4\mathbf{a}_z = 2\mathbf{a}_y + 4\mathbf{a}_z$$

(b) The distance vector from P to Q

$$\begin{aligned}\vec{\mathbf{r}}_{PQ} &= \vec{\mathbf{r}}_Q - \vec{\mathbf{r}}_P \\ &= (-3,1,5) - (0,2,4) \\ &= -3\mathbf{a}_x - \mathbf{a}_y + \mathbf{a}_z\end{aligned}$$

Solution to Example 3

(c)The distance between P and Q

Since $\vec{\mathbf{r}}_{PQ}$ is a distance vector, the distance between P and Q is the magnitude of this distance vector.

$$d = \left|\mathbf{r}_{PQ}\right| = \sqrt{(-3)^2 + (-1)^2 + (1)^2} = 3.317$$

Example 4

Given points $M(-1,2,1)$ and $N(3,-3,0)$, find the vector R_{MN} and the unit vector a_{MN}.

Solution

$$\mathbf{R}_{MN} = (3\mathbf{a}_x - 3\mathbf{a}_y + 0\mathbf{a}_z) - (-1\mathbf{a}_x + 2\mathbf{a}_y + 1\mathbf{a}_z)$$

$$= \underline{\underline{4\mathbf{a}_x - 5\mathbf{a}_y - \mathbf{a}_z}}$$

$$\mathbf{a}_{MN} = \frac{\mathbf{R}_{MN}}{\left|\mathbf{R}_{MN}\right|} = \frac{4\mathbf{a}_x - 5\mathbf{a}_y - 1\mathbf{a}_z}{\sqrt{4^2 + (-5)^2 + (-1)^2}}$$

$$= \underline{\underline{0.617\mathbf{a}_x - 0.772\mathbf{a}_y - 0.154\mathbf{a}_z}}$$

Multiplication of Vectors

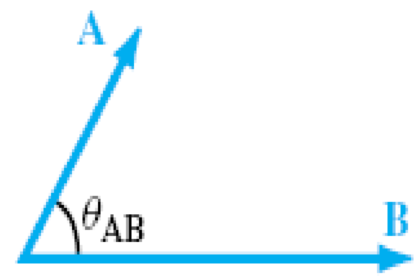
- When two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are multiplied, the result is either a scalar or vector, depending on how they are multiplied.

- Two types of multiplication:
  - Scalar (or dot) product $\vec{\mathbf{A}} \bullet \vec{\mathbf{B}}$
  - Vector (or cross) product $\vec{\mathbf{A}} \times \vec{\mathbf{B}}$

The Dot Product

- Given two vectors A and B, the *dot product*, or *scalar product*, is defines as the prouct of the magnitude of A, the magnitude of B, and the cosine of the smaller angle between them:

$$\boxed{\mathbf{A}\cdot\mathbf{B} = |\mathbf{A}||\mathbf{B}|\cos\theta_{AB}}$$



- The dot product is a scalar, and it obeys the commutative law:

$$\boxed{\mathbf{A}\cdot\mathbf{B} = \mathbf{B}\cdot\mathbf{A}}$$

– Distributive Law

$$\boxed{\vec{\mathbf{A}}\bullet\left(\vec{\mathbf{B}}+\vec{\mathbf{C}}\right) = \vec{\mathbf{A}}\bullet\vec{\mathbf{B}}+\vec{\mathbf{A}}\bullet\vec{\mathbf{C}}}$$

$$\vec{\mathbf{A}}\bullet\vec{\mathbf{A}} = A^2 = |\mathbf{A}|^2$$

Dot Product in Cartesian

- The two vectors, $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are said to be perpendicular or **orthogonal** (90°) with each other if;

$$\vec{\mathbf{A}} \bullet \vec{\mathbf{B}} = 0$$

Properties of dot product of unit vectors:

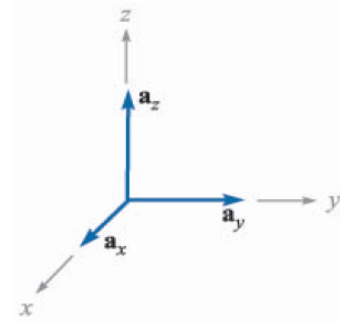
$$a_x \bullet a_x = a_y \bullet a_y = a_z \bullet a_z = 1$$

$$a_x \bullet a_y = a_y \bullet a_z = a_z \bullet a_x = 0$$

Dot Product in Cartesian

- For any vector $\mathbf{A} = A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z$ and

$$\mathbf{B} = B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z$$



Since the angle between two unit vectors of the Cartesian coordinate system is 90^0, we then have:

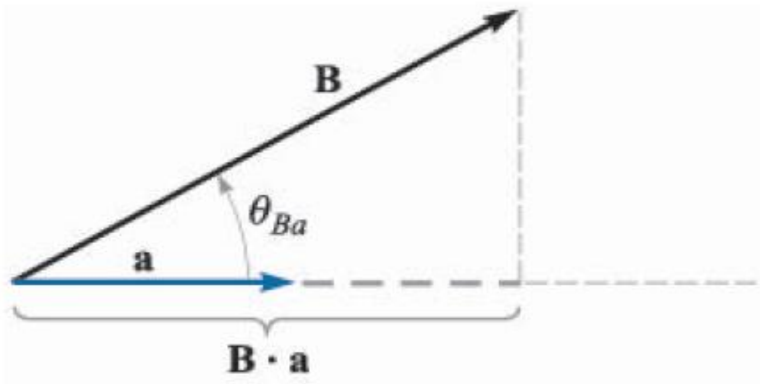
$$\mathbf{a}_x \bullet \mathbf{a}_y = \mathbf{a}_y \bullet \mathbf{a}_x = \mathbf{a}_x \bullet \mathbf{a}_z = \mathbf{a}_z \bullet \mathbf{a}_x = \mathbf{a}_y \bullet \mathbf{a}_z = \mathbf{a}_z \bullet \mathbf{a}_y = 0$$

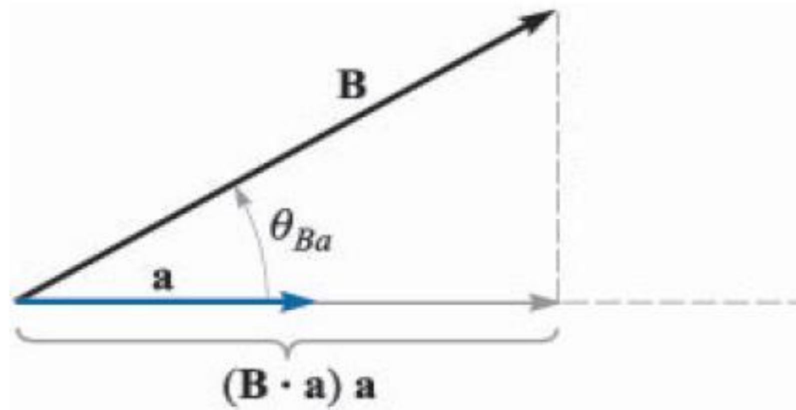
- And thus, only three terms remain, giving finally:

$$\boxed{\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z}$$

The Dot Product

- One of the most important applications of the dot product is that of finding the component of a vector in a given direction.





- The scalar component of B in the direction of the unit vector a is B·a
- The vector component of B in the direction of the unit vector a is (B·a)a
- The geometrical term projection is also used with the dot product. Thus, B·a is the projection of B in the a direction.

$$\boxed{\mathbf{B}\cdot\mathbf{a} = |\mathbf{B}||\mathbf{a}|\cos\theta_{Ba} = |\mathbf{B}|\cos\theta_{Ba}}$$

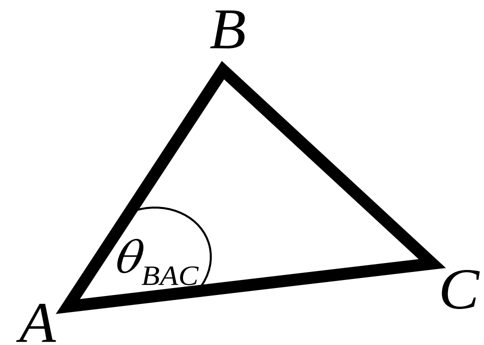
The Dot Product

<u>Example</u>

The three vertices of a triangle are located at $A(6,–1,2)$, $B(–2,3,–4)$, and $C(–3,1,5)$. Find: (*a*) R_{AB}; (*b*) R_{AC}; (*c*) the angle θ_{BAC} at vertex *A*; (*d*) the vector projection of R_{AB} on R_{AC}.

$$\mathbf{R}_{AB} = (-2\mathbf{a}_x + 3\mathbf{a}_y - 4\mathbf{a}_z) - (6\mathbf{a}_x - \mathbf{a}_y + 2\mathbf{a}_z) = \underline{\underline{-8\mathbf{a}_x + 4\mathbf{a}_y - 6\mathbf{a}_z}}$$

$$\mathbf{R}_{AC} = (-3\mathbf{a}_x + 1\mathbf{a}_y + 5\mathbf{a}_z) - (6\mathbf{a}_x - \mathbf{a}_y + 2\mathbf{a}_z) = \underline{\underline{-9\mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z}}$$



$$\mathbf{R}_{AB} \cdot \mathbf{R}_{AC} = |\mathbf{R}_{AB}||\mathbf{R}_{AC}|\cos\theta_{BAC}$$

$$\Rightarrow \cos\theta_{BAC} = \frac{\mathbf{R}_{AB} \cdot \mathbf{R}_{AC}}{|\mathbf{R}_{AB}||\mathbf{R}_{AC}|} = \frac{(-8\mathbf{a}_x + 4\mathbf{a}_y - 6\mathbf{a}_z)\cdot(-9\mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z)}{\left|\sqrt{(-8)^2 + (4)^2 + (-6)^2}\right|\left|\sqrt{(-9)^2 + (2)^2 + (3)^2}\right|} = \frac{62}{\left|\sqrt{116}\right|\left|\sqrt{94}\right|} = 0.594$$

$$\Rightarrow \theta_{BAC} = \cos^{-1}(0.594) \quad = \underline{\underline{53.56^\circ}}$$

The Dot Product

<u>Example</u>

The three vertices of a triangle are located at $A(6,-1,2)$, $B(-2,3,-4)$, and $C(-3,1,5)$. Find: (*a*) R_{AB}; (*b*) R_{AC}; (*c*) the angle θ_{BAC} at vertex *A*; (*d*) the vector projection of R_{AB} on R_{AC}.

$$\mathbf{R}_{AB} \text{ on } \mathbf{R}_{AC} = \left(\mathbf{R}_{AB}\cdot\mathbf{a}_{AC}\right)\mathbf{a}_{AC}$$

$$= \left((-8\mathbf{a}_x + 4\mathbf{a}_y - 6\mathbf{a}_z)\frac{(-9\mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z)}{\left|\sqrt{(-9)^2+(2)^2+(3)^2}\right|} \right) \frac{(-9\mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z)}{\left|\sqrt{(-9)^2+(2)^2+(3)^2}\right|}$$

$$= \frac{62}{\sqrt{94}}\,\frac{(-9\mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z)}{\sqrt{94}}$$

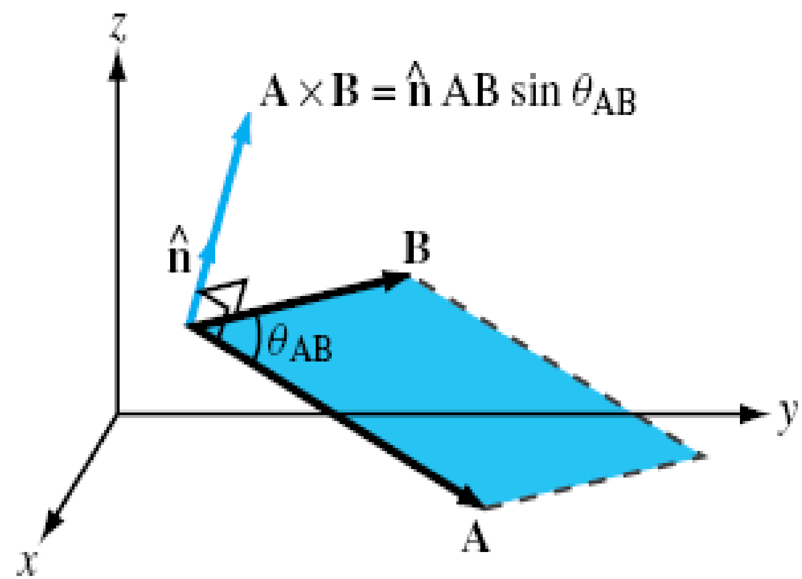
$$= \underline{\underline{-5.963\mathbf{a}_x + 1.319\mathbf{a}_y + 1.979\mathbf{a}_z}}$$

The Cross Product

Given two vectors A and B, the magnitude of the *cross product*, or *vector product*, written as A×B, is defined as the product of the magnitude of A, the magnitude of B, and the sine of the smaller angle between them.

$$\boxed{\mathbf{A}\times\mathbf{B}=\mathbf{a}_N\,|\mathbf{A}||\mathbf{B}|\sin\theta_{AB}}$$

Direction of a_N is perpendicular (90°) to the plane containing A and B



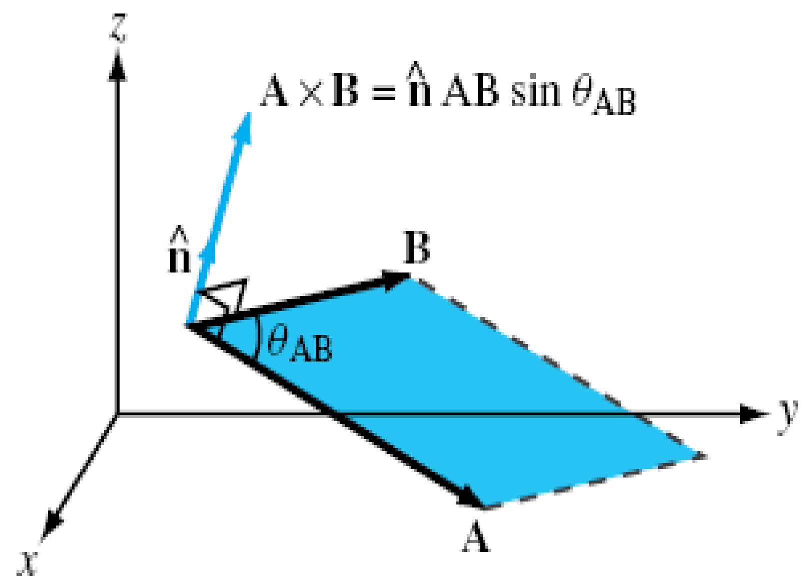
(a) Cross product

The Cross Product

The direction of A×B is perpendicular to the plane containing A and B and is in the direction of advance of a right-handed screw as A is turned into B.

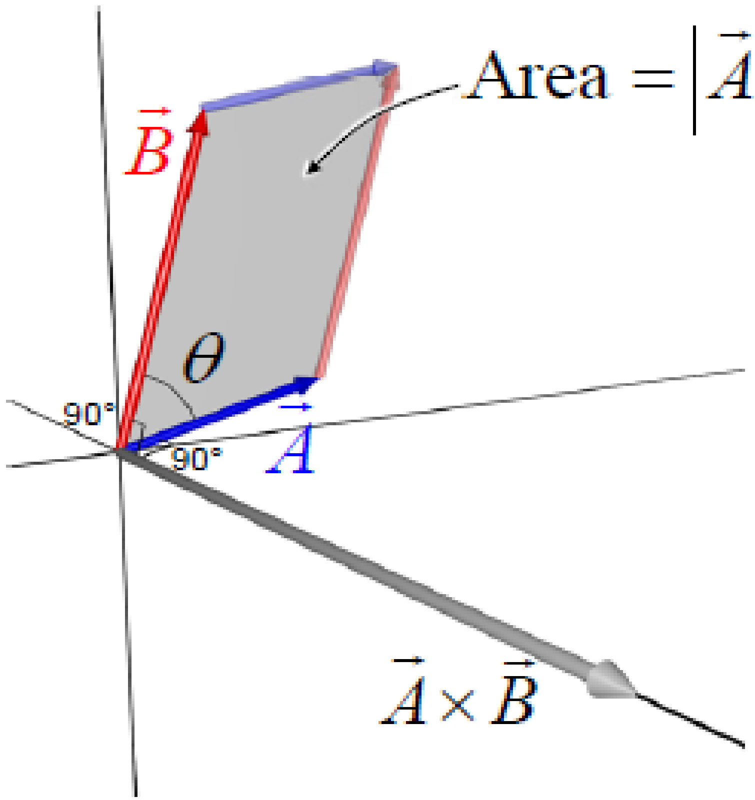
$$\boxed{\mathbf{A} \times \mathbf{B} = \mathbf{a}_N \left|\mathbf{A}\right|\left|\mathbf{B}\right| \sin\theta_{AB}}$$

Direction of a_N is perpendicular (90°) to the plane containing A and B



(a) Cross product

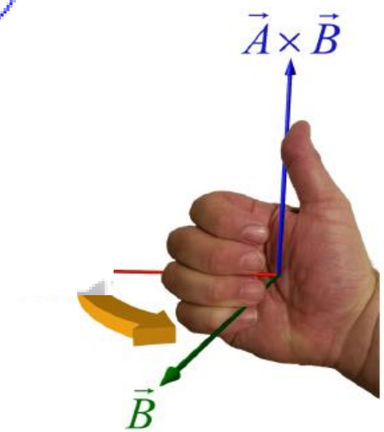
The cross product is all about area and calculating vectors that are perpendicular to $\vec{A}$ and $\vec{B}$.



$$\text{Area} = \left|\vec{A}\times\vec{B}\right|$$

$$\vec{A}\times\vec{B} = \left|\vec{A}\right|\left|\vec{B}\right|\sin\theta\,\hat{a}_n$$

$\hat{a}_n$ is a unit vector perpendicular to the plane defined by $\vec{A}$ and $\vec{B}$.



The Cross Product

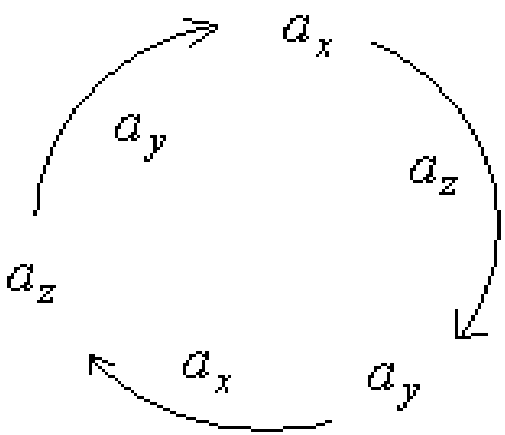
- Cross product obeys the following:
  - It is not commutative $$(\mathbf{B}\times\mathbf{A}) = -(\mathbf{A}\times\mathbf{B})$$
  - It is not associative $$\vec{\mathbf{A}}\times\left(\vec{\mathbf{B}}\times\vec{\mathbf{C}}\right) \neq \left(\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right)\times\vec{\mathbf{C}}$$
  - It is distributive $$\vec{A}\times\left(\vec{B}+\vec{C}\right) = \vec{A}\times\vec{B}+\vec{A}\times\vec{C}$$

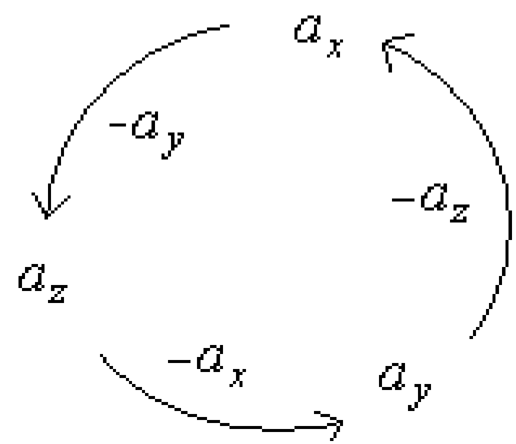
Properties of Vector Product

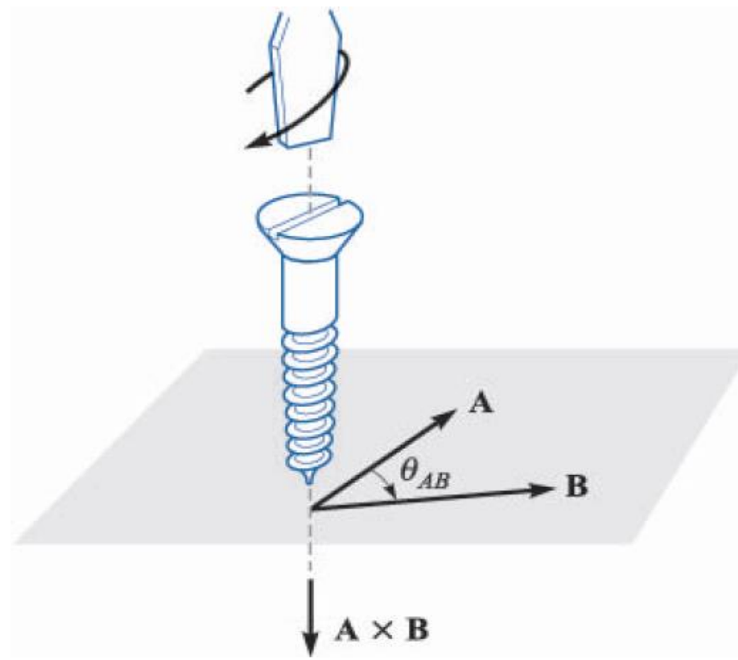
- Properties of cross product of unit vectors:

$$\mathbf{a}_x \times \mathbf{a}_y = \mathbf{a}_z$$
$$\mathbf{a}_y \times \mathbf{a}_z = \mathbf{a}_x$$
$$\mathbf{a}_z \times \mathbf{a}_x = \mathbf{a}_y$$

Or by using cyclic permutation:







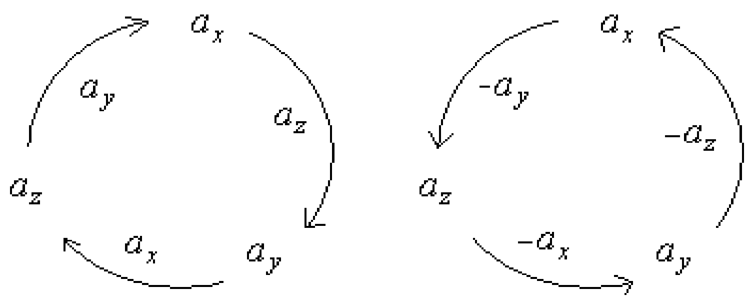
Cross product in Cartesian

- The cross product of two vectors of Cartesian coordinate:

$$\vec{\mathbf{A}} = A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z$$

$$\vec{\mathbf{B}} = B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z$$

yields the sum of nine simpler cross products, each involving two unit vectors.



Cross product in Cartesian

By using the properties of cross product, it gives

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \left(A_y B_z - A_z B_y\right)\mathbf{a}_x + \left(A_z B_x - A_x B_z\right)\mathbf{a}_y + \left(A_x B_y - A_y B_x\right)\mathbf{a}_z$$

and be written in more easily remembered form:

$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

The Cross Product

Example

Given A = 2a_x–3a_y+a_z and B = –4a_x–2a_y+5a_z, find A×B.

$$\mathbf{A}\times\mathbf{B} = (A_yB_z - A_zB_y)\mathbf{a}_x + (A_zB_x - A_xB_z)\mathbf{a}_y + (A_xB_y - A_yB_x)\mathbf{a}_z$$

$$= \big((-3)(5)-(1)(-2)\big)\mathbf{a}_x + \big((1)(-4)-(2)(5)\big)\mathbf{a}_y + \big((2)(-2)-(-3)(-4)\big)\mathbf{a}_z$$

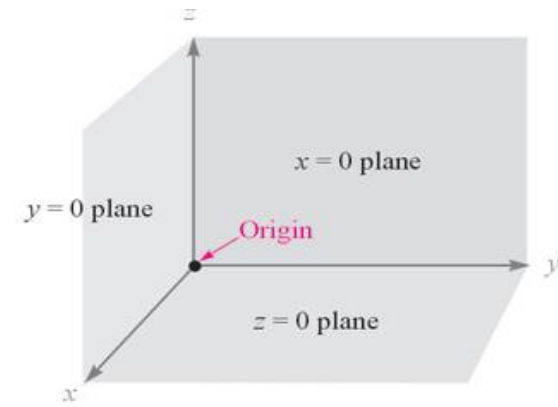
$$= \underline{\underline{-13\mathbf{a}_x - 14\mathbf{a}_y - 16\mathbf{a}_z}}$$

Coordinate Systems

Coordinate Systems

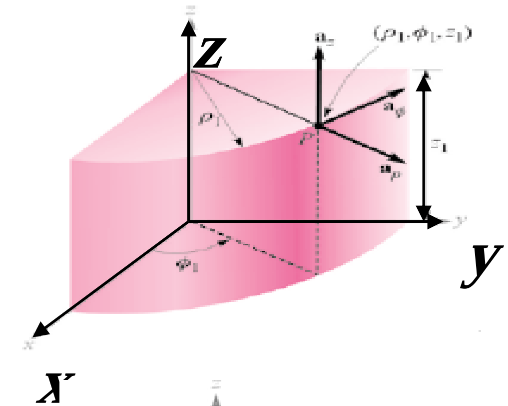
- Cartesian coordinates:

$$(x, y, z)$$



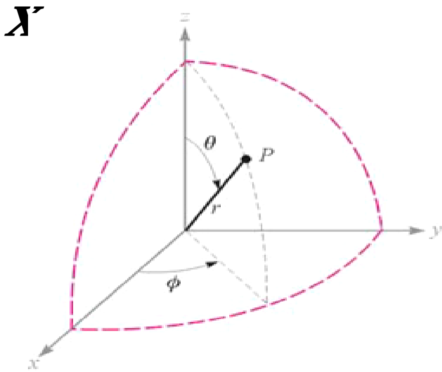
- Circular Cylindrical coordinates:

$$(\rho, \phi, z)$$



- Spherical coordinates:

$$(r, \theta, \phi)$$



1.2 Coordinate Systems

- In Electromagnetics, all the quantities are functions of space and time. In order to describe the spatial. variations of the quantities, all the points in space must be defined using an appropriate co-ordinate system.
- Although there are different co-ordinate systems available, the three well-known co-ordinate systems are Cartesian (or rectangular) co-ordinates, circular cylindrical co-ordinates and spherical coordinates.

1.2.1 The Cartesian or Rectangular Coordinate System

- A point 'P' is represented in Cartesian or rectangular co-ordinates as (X, Y, Z).
- From Fig. 1.1, it is known that any point in rectangular co-ordinate is the intersection of three plane (i) constant X-plane (ii) constant Y-plane and (iii) constant Z-plane which are mutually perpendicular. The range of the coordinate variables x, y and z are $-\infty < x < \infty$, $-\infty < y < \infty$ and $-\infty < z < \infty$.
- A vector in Cartesian co-ordinates a vector $\vec{r}$ can be written as (R_x, R_y, R_z) or

$$\vec{r} = x\,\hat{a}_x + y\,\hat{a}_y + z\,\hat{a}_z \quad or \quad \vec{r} = x\hat{x} + y\hat{y} + z\hat{z} \qquad (1.1)$$

where $\hat{a}_x,\ \hat{a}_y\ and\ \hat{a}_z$ or $\hat{x}, \hat{y}\ and\ \hat{z}$

are unit vectors in the directions of x, y and z.

1.2 Coordinate Systems (Continued)

1.2.1 The Cartesian or Rectangular Coordinate System

In A right-handed coordinate system:

- The thump identify the x axis direction.
- The forefinger identify the y axis direction.
- The middle identify the z axis direction.

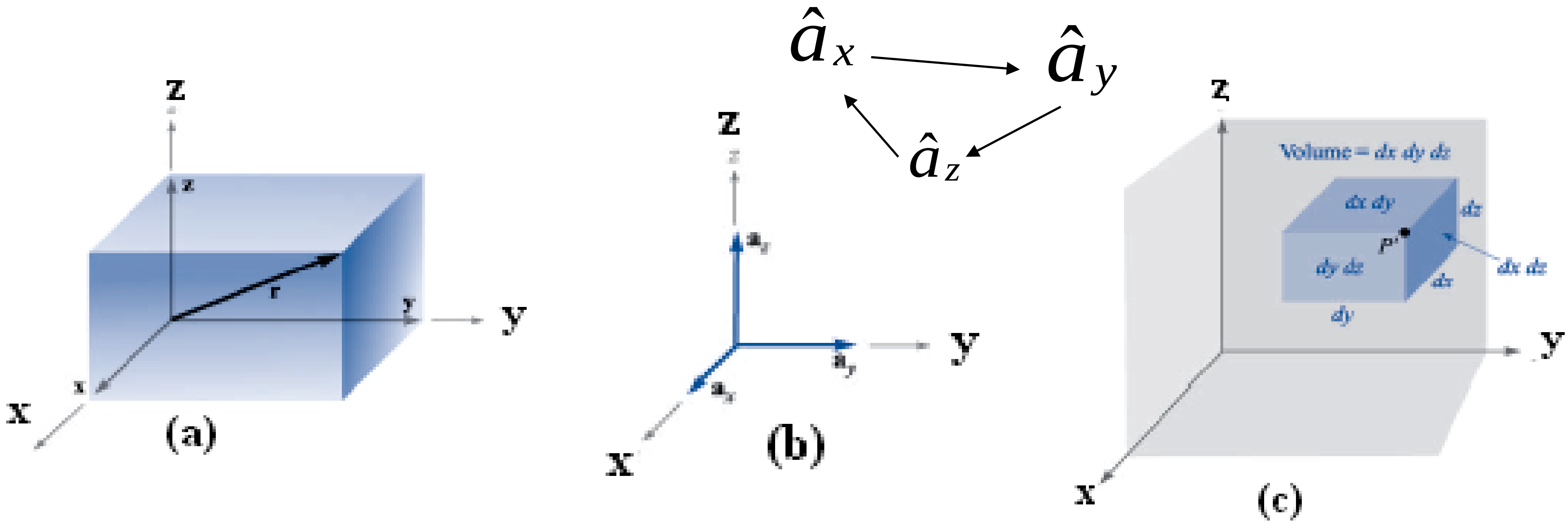


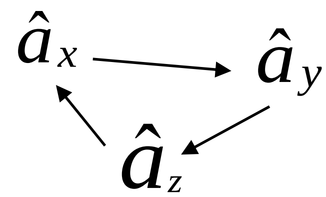
Fig. 1.1 A right-handed rectangular coordinate system. (a) The component vectors x, y and z of a vector r. (b) The unit vectors. (c) The differential volume elements dx, dy, and dz.

1.2 Coordinate Systems (Continued)

1.2.1 The Cartesian or Rectangular Coordinate System (Continued)

- If we visualize three planes by intersecting at general point P as shown in Fig. 1.1(c), whose coordinates x, y and z, we may increase each coordinate by a differential amount and obtain three slightly displaced planes. We define the following quantities:

1. The differential length vector $d\vec{l}$ is given by:



$$d\vec{l} = dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z \qquad (1.2)$$

2. The differential area vectors $d\vec{s}$ are given by :

$$d\vec{S}_1 = dxdy\hat{a}_z$$
$$d\vec{S}_2 = dydz\hat{a}_x \qquad (1.3)$$
$$d\vec{S}_3 = dzdx\hat{a}_y$$

3. The differential volume is dv (scalar value) given by :

$$dv = dxdydz \qquad (1.4)$$

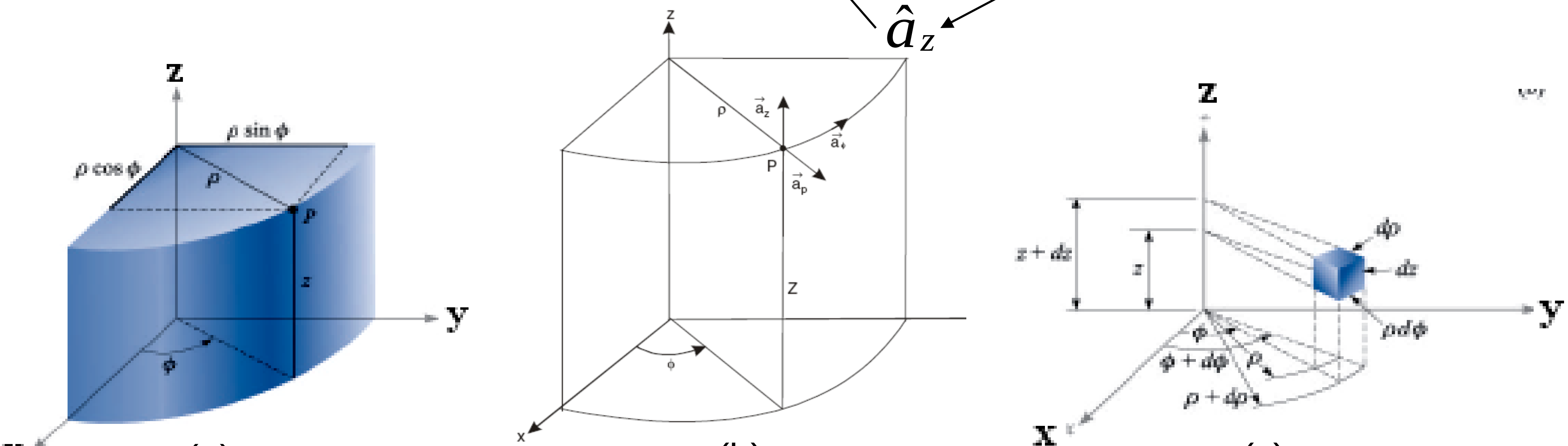
1.2 Coordinate Systems (Continued)

1.2.2 The Cylindrical Coordinate System

- The Cylindrical coordinate system (ρ, ϕ, z) is shown in Fig. 1.2. A vector $\vec{r}$ is represented in this system as:

$$\vec{r} = \rho\,\hat{a}_\rho + z\,\hat{a}_z \qquad (1.5)$$

$$\hat{a}_\rho \rightarrow \hat{a}_\phi \rightarrow \hat{a}_z \rightarrow \hat{a}_\rho$$



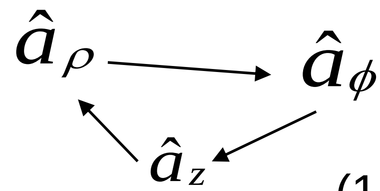
(a) (b) (c)

Fig. 1.2 A right-handed cylindrical coordinate system. (a) The relationship between rectangular variables x, y, z and the cylindrical coordinate variables ρ, ϕ, z. (b) The unit vectors. (c) The differential volume elements d ρ, ρ d ϕ and dz.

1.2 Coordinate Systems (Continued)

1.2.2 The Cylindrical Coordinate System (Continued)

- A differential volume element in cylindrical coordinate may be obtained by increasing ρ, ϕ and z by the differential increments dρ, dϕ and dz as shown in Fig. 1.2(c). We define the following quantities:



1. The differential length vector $d\vec{l}$ is given by:

$$d\vec{l} = d\rho\,\hat{a}_\rho + \rho d\phi\,\hat{a}_\phi + dz\,\hat{a}_z \qquad (1.6)$$

2. The differential area vectors $d\vec{s}$ are given by:

$$d\vec{S}_1 = d\rho\rho d\phi\,\hat{a}_z$$

$$d\vec{S}_2 = \rho d\phi dz\,\hat{a}_\rho \qquad (1.7)$$

$$d\vec{S}_1 = dz d\rho\,\hat{a}_\phi$$

3. The differential volume is dv (scalar value) and given by:

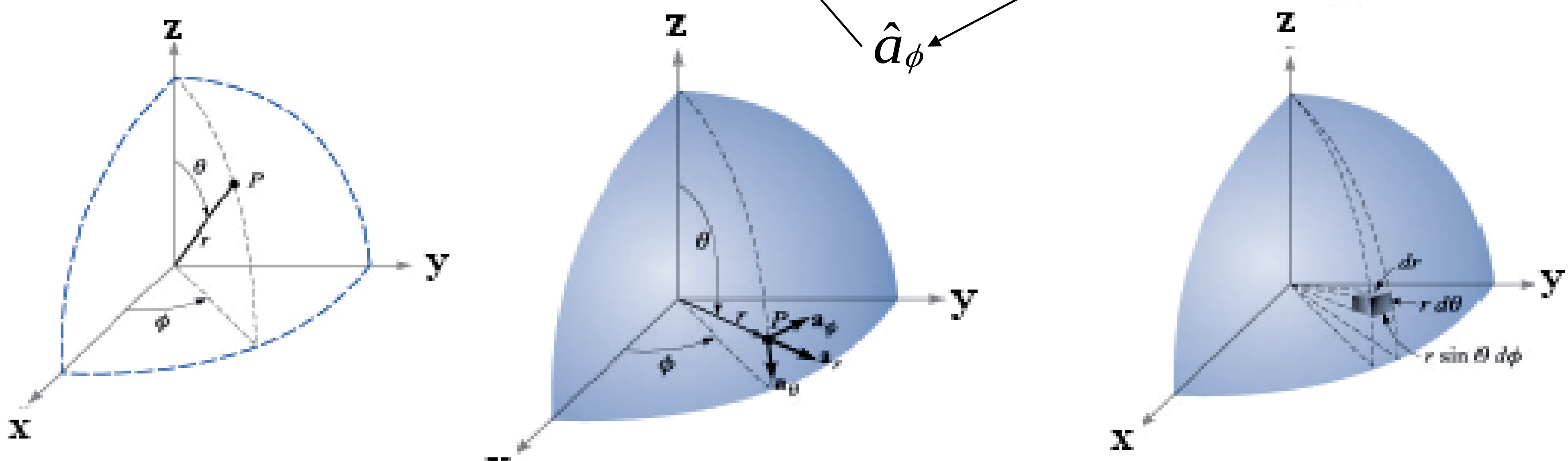
$$dv = \rho d\rho d\phi dz \qquad (1.8)$$

1.2 Coordinate Systems (Continued)

1.2.3 The Spherical Coordinate System

- The spherical coordinate system (r, θ, ϕ) is shown in Fig. 1.3. A vector $\vec{r}$ is represented in this system as:

$$\vec{r} = r\,\hat{a}_r \qquad \hat{a}_r \rightarrow \hat{a}_\theta \rightarrow \hat{a}_\phi \rightarrow \hat{a}_r \tag{1.9}$$



(a) (b) (c)

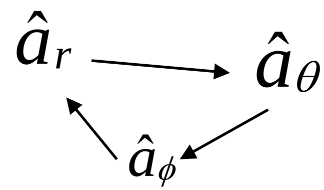
Fig. 1.3 A right-handed spherical coordinate system. (a) Three spherical coordinate variables r, θ and ϕ. (b) The unit vectors. (c) The differential volume element.

1.2 Coordinate Systems (Continued)

1.2.3 The Spherical Coordinate System (Continued)

- A differential volume element in spherical coordinate may be obtained by increasing r, θ, and ϕ by the differential increments dr, dθ and dϕ as shown in Fig. 1.3(c). We define the following quantities:

1. The differential length vector $d\vec{l}$ is given by:



$$d\vec{l} = dr\,\hat{a}_r + r d\theta\,\hat{a}_\theta + r\sin\theta d\phi\,\hat{a}_\phi \qquad (1.10)$$

2. The differential area vectors $d\vec{s}$ are given by:

$$d\vec{S}_1 = r dr d\theta\,\hat{a}_\phi$$

$$d\vec{S}_2 = r^2 \sin\theta d\theta d\phi\,\hat{a}_r \qquad (1.11)$$

$$d\vec{S}_1 = r\sin\theta d\phi dr\,\hat{a}_\theta$$

3. The differential volume is dv (scalar value) given by:

$$dv = r^2 \sin\theta dr d\theta d\phi \qquad (1.12)$$

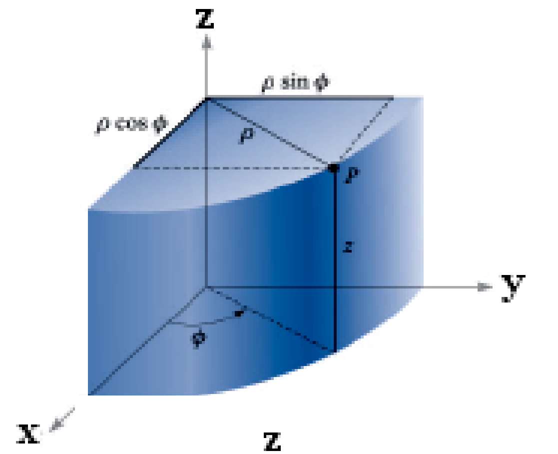
1.2 Coordinate Systems (Continued)

1.2.4 Relationship between the Rectangular and Cylindrical Coordinate Systems

(a) The Variables (x, y and z) versus (ρ, ϕ, z)

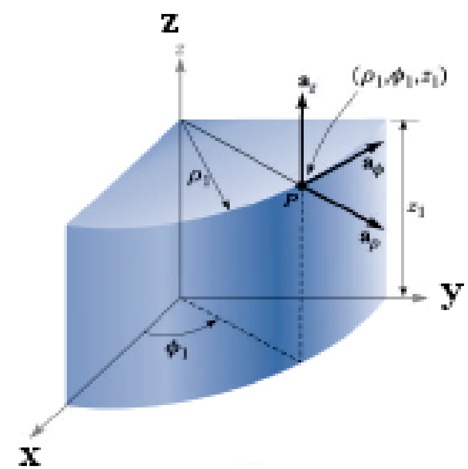
- As shown in Fig. 1.2, the variables of rectangular coordinate system (x, y and z) can be written as functions of that of cylindrical coordinate systems (ρ, ϕ and z) as follows:

$$x = \rho \cos\phi$$
$$y = \rho \sin\phi \qquad (1.13)$$
$$z = z$$



- From the other viewpoint, we may express cylindrical variables (ρ, ϕ and z) in terms of (x, y and z) as follov

$$\rho = \sqrt{x^2 + y^2}$$
$$\phi = \tan^{-1}\frac{y}{x} \qquad (1.14)$$
$$z = z$$



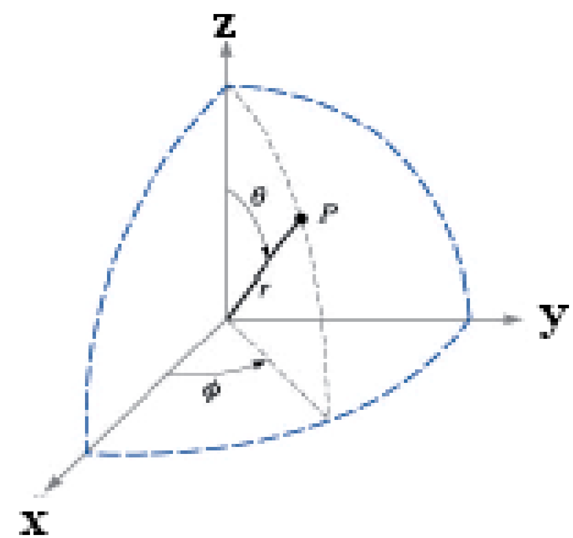
1.2 Coordinate Systems (Continued)

1.2.5 Relationship between the Rectangular and Spherical Coordinate Systems

(a) The Variables (x, y and z) versus (r, θ, ϕ)

- As shown in Fig. 1.3, the variables of rectangular coordinate system (x, y and z) can be written as functions of that of spherical coordinate systems (r, θ and ϕ) as follows:

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi \qquad (1.16)$$
$$z = r\cos\theta$$



- From the other viewpoint, we may express cylindrical variables (r, θ and ϕ) in terms of (x, y and z) as follows:

$$r = \sqrt{x^2 + y^2 + z^2}$$
$$\theta = \tan^{-1}\frac{\sqrt{x^2 + y^2}}{z} \qquad (1.14)$$
$$\phi = \tan^{-1}\frac{y}{x}$$

Thank you for your attention

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